Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

End Semester Examinations - 2015-16 Even Semester - May 2016

15MA3005 Complex Analysis

Set B **Time : 3 hrs**
Total Marks: 100

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1. (a). If $z = x+iy$ and $f(z) = x-iy$ show that f is nowhere differentiable. (marks 7)
(b). Find an analytic function $f = u + iv$ with $u(x,y) = x^2 - y^2$. (marks 7)
(c). State and prove the uniqueness theorem on power series. (marks 6)

OR

2. (a). Define the trigonometric and hyperbolic functions. (marks 10)
(b). Show that sinz and cosz are unbounded in the complex plane. (marks 10)

3. (a). Let f be a complex-valued function whose domain contains [a,b], prove that
$\left|\int_a^b f(t)dt\right| \le \int_a^b |f(t)|dt$. (marks 10)
(b). If f is entire, prove that f is everywhere the derivative of an analytic function.
(marks 10)

OR

4. (a). Let f be analytic inside and on a positively oriented contour γ. Prove that, if a is in the interior of γ, $f(a) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-a}dz$. (marks 10)
(b). Let γ be a circle $|z-2i| = 4$, evaluate $\int_\gamma \frac{z}{z^2+9}dz$. (marks 10)

5. (a). If f is a bonded entire function, prove that f is constant. (marks 10)
(b). Let p be a polynomial function of degree at least 1. Prove that, $p(z) = 0$ for at least one z. (marks 10)

OR

6. Let f be analytic within the annulus $A = \{z : R_1 < |z-a| < R_2\}$. Prove that f has the series representation $f(z) = \sum_{k=-\infty}^{\infty} c_k (z-a)^k$ valid for $z \in A$. The coefficients c_k are given by $c_k = \frac{1}{2\pi i}\int_c \frac{f(w)}{(w-a)^{k+1}}dw,\ k = 0, \pm1, \pm2......$ where c is the simple closed curve that lies entirely within A and has a in its interior.

7. Evaluate $\int_\Gamma \frac{z+3}{(z-1)(z-2)(z+4)}dz$ where Γ is the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$.

OR

8. Let f be analytic and non constant in a domain D of the complex plane. Suppose that $f'(z_0) = f''(z_0) = ...f^{(n-1)}(z_0) = 0,\quad f^{(n)}(z_0) \ne 0,\quad z_0 \in D$. Prove that, the mapping $z \to f(z)$ magnifies n times the angle between two intersecting differentiable arcs that meet at z_0.

9. (a). Prove that the bilinear transformation $f(z) = \frac{az+b}{cz+d}, ad-bc \neq 0$ has at most two fixed points. (marks 7)

(b). Define cross ratio and prove that it is invariant under bilinear transformation.

(marks 7)

(c). If $w = f(z)$ is a unique bilinear transformation that maps the distinct points z_1, z_2 and z_3 onto the three distinct points w_1, w_2 and w_3 respectively, prove that

$$\frac{z-z_1}{z-z_3}\frac{z_2-z_3}{z_2-z_1} = \frac{w-w_1}{w-w_3}\frac{w_2-w_3}{w_2-w_1}.$$ (marks 6)

Wishing you All the Best
